7

Modelling linear relationships

Syllabus reference

Algebraic modelling 2

- Modelling linear relationships

In this chapter we will begin to look at modelling. We will explore situations that can be modelled by a linear function and what the features of the linear graph mean about the situation being modelled.

ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest Preliminary Course* eBookPLUS or ask your teacher for a copy.

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Digital doc
SkillSHEET 7.1
doc-1523
Recognising linear relationships

Recognising linear functions

1 Which of the functions below are linear functions?

a $y = x^2$ **b** $y = 2x$ **c** $y = \frac{x}{2}$ **d** $y = \frac{2}{x}$

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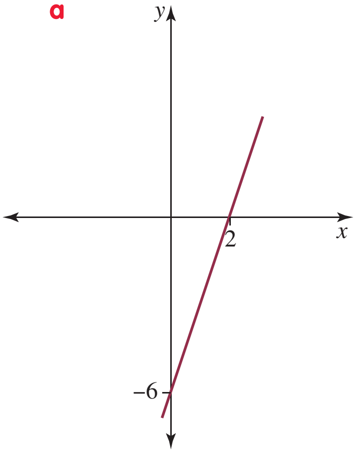
SkillSHEET 7.2
doc-1529
Gradient of a straight line

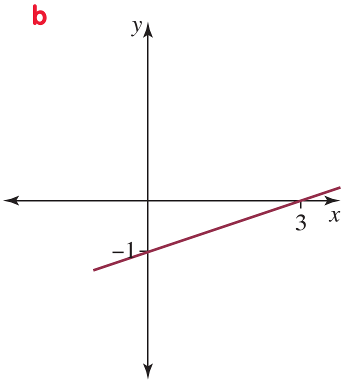
Gradient of a straight line

2 Find the gradient of each of the following.

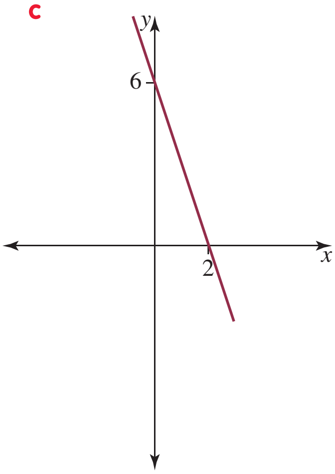
a



b



c



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Substitution

Substitution

3 Complete the following.

a Given that $y = 5 - 2x$, find the value of y when $x = 6$.

b Given that $y = 4x + 7$, find the value of y when $x = -3$.

c Given that $y = \frac{10 - 2x}{5}$, find the value of y when $x = 0$.

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Graphing linear equations

Graphing linear equations

4 Draw the graph of:

a $y = x + 1$ **b** $y = 2x - 4$ **c** $y = 6 - 2x$.

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Solving linear equations

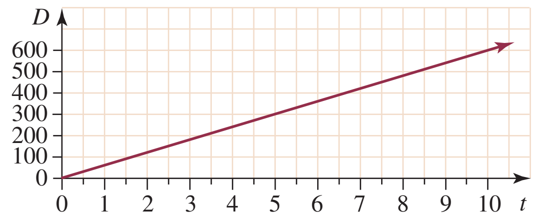
Solving linear equations

5 Solve each of the following equations.

a $2x - 8 = 0$ **b** $12 - 4x = 0$ **c** $5x - 2 = 0$

7A Graphing linear functions

Imagine a car travelling at a constant speed of 60 km/h. The graph at right compares the distance travelled with time.



This graph can be given by the **relation** $D = 60t$. This relation is an example of a function. A **function** is a rule with two variables. In the above example time, t, is the **independent variable**. This is the variable for which we can substitute any value. Distance, D, is the **dependent variable** as its value depends on the value substituted for t.

A **linear function** is a graph that, when drawn, is represented by a straight line. Linear functions are drawn from a table of values. The independent variable is graphed on the horizontal axis and the dependent variable is graphed on the vertical axis.

WORKED EXAMPLE 1

The table below shows the amount of money earned by a wage earner.

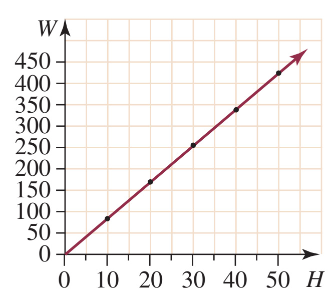
Hours (H)	10	20	30	40	50
Wage (W)	85	170	255	340	425

Draw the graph of wage, W, against hours, H.

THINK

1. Draw the graph with H on the horizontal axis and W on the vertical axis.
2. Plot the points (10, 85) (20, 170) (30, 255) (40, 340) and (50, 425).
3. Join the points with a straight line.

DRAW



In many examples we are required to draw a graph from an algebraic rule. In such an example we need to create our own table. To do this, we can choose any sensible value to use for the independent variable.

WORKED EXAMPLE 2

The conversion of Australian dollars, A, to US dollars, U, can be given by the rule $U = 0.8A$. Draw the graph of this function.

THINK

WRITE/DRAW

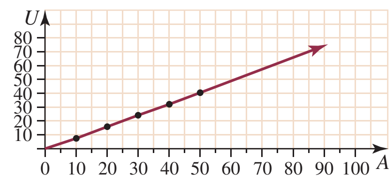
Method 1

1. Draw a table choosing several values to substitute for the independent variable, A.

2 Calculate the value of U for each value of A in the table.

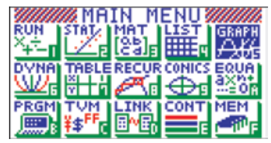
A	10	20	30	40	50
U	8	16	24	32	40

3 Draw the axes, plot the points generated and join each point with a straight line, extending the line as required.



Method 2

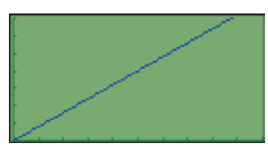
1 From the **MENU** select **GRAPH**.



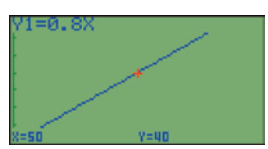
2 Delete any existing functions and enter the rule. The calculator always uses **Y** as the dependent variable and **X** as the independent variable. Enter **Y1** = **0.8X**. Be sure to use the [**X,θ,T**] button for **X**. Finish by pressing [EXE].

3 Next we need to rule up our coordinate axes. This is done using the V-Window function. Press [SHIFT] [F3]. This sets the minimum and maximum values for both **X** and **Y** as well as the increments on each axis. Enter the settings shown on the screen at right, which replicate the axes that are drawn in the worked example.

4 Press [EXE] to return to the previous screen and then [F6] to draw the graph.



5 To see the points on the graph use the Trace function. Press [SHIFT] [F1] and use your arrow keys to see the points drawn.



When applying a function we need to understand the idea of:

input ⟶ process ⟶ output.

The independent variable is the *input*, a calculation is made which is the *process* and the *output* is the value of the dependent variable.

An independent variable is substituted (input), a calculation is made (process) according to the rule defined by the function and the dependent variable (output) is the result.

WORKED EXAMPLE 3

A preschool has hired an entertainment group to entertain their children at a concert. The cost of staging the concert is given by the function $C = 80 + 3n$, where C is the cost and n is the number of children attending the concert.

Draw the graph of this function.

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Worked example 3

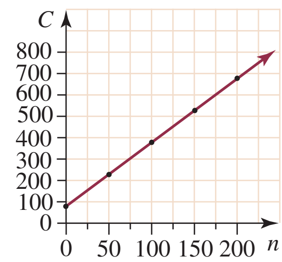
THINK | **WRITE/DRAW**

1 Draw a table choosing five values for n to substitute.

2 Calculate the values of C for each value of n chosen.

n	0	50	100	150	200
C	80	230	380	530	680

3 Draw the axes, plot the points generated and join with a straight line.



REMEMBER

1. A function is a rule for a calculation that consists of an independent and dependent variable.
2. Values are substituted for the independent variable and a value for the dependent variable is generated.
3. A linear function is represented by a straight line when graphed.
4. When graphing a linear function, the independent variable is shown on the horizontal axis and the dependent variable on the vertical axis.
5. To graph a linear function we draw up a table of values, plot the points generated by that table then join these points with a straight line.

EXERCISE 7A Graphing linear functions

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Recognising linear relationships

1 WE1 The table below shows the amount of money, M, earned for delivering a number of pamphlets, P, to letterboxes.

P	1000	2000	3000	4000	5000
M	50	100	150	200	250

Draw the graph of this function.

2 Use the graph drawn in question 1 to find the amount of money earned by a person delivering:

a 8000 pamphlets

b 9500 pamphlets.

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Spreadsheet
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Graph paper

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GC program — Casio
doc-1525
Linear

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GC program — TI
doc-1526
Linear

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Myrule

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GC program — TI
doc-1528
Myrule

3 Australian dollars can be converted to Japanese yen using the algebraic rule $Y = 80A$. To draw a conversion graph, the table below is used.

A	100	200	300	400	500
Y	8000	16 000	24 000	32 000	40 000

Draw a graph converting Australian dollars to Japanese yen.

4 WE2 The cost of an international telephone call can be given by the rule $C = 1.5t$, where C is the cost of the call and t is the length of the call in minutes. Draw a graph showing the cost of a telephone call.

5 Use the graph from question **4** to calculate the cost of a telephone call that lasts for:
a 17 minutes
b 45 minutes.

6 The distance, d, travelled by a cyclist can be given by the algebraic rule $d = 15t$, where t is the time in hours that the cyclist has been riding. Draw a graph showing the distance travelled by the cyclist against time.

7 WE3 The cost, C, of a taxi journey can be given by the rule $C = 4 + 1.5d$, where d is the distance of the journey in kilometres. Draw a graph showing the cost of a taxi journey.

8 A tree bought as a seedling is 80 cm tall. It then grows at an average rate of 12 cm per year. Draw a graph that will show the height of the tree each year.

9 Casey has a job that pays $10 per hour. Draw a graph that will show the money earned against hours worked.

10 It costs Bill $1850 per week to operate his business developing photographs. Bill charges $8 to develop a roll of film. Draw a graph that shows the profit or loss he makes against the number of rolls of film developed.

Further development

11 The cost of hiring a tennis court consists of a booking fee and an hourly rate.

a Write an equation for the total cost in terms of the hourly rate.
b Sketch a graph of this relationship.
c What would be the charge for 3 hours?

12 A singing telegram service charges a $60 appearance fee and $8 per minute sung.
a Write and equation for the total cost of a singing telegram in terms of the number of minutes sung.
b Sketch a graph of the relationship.
c What would be the charge for a 5 minute singing telegram?
d How many minutes can be bought with $150? (Whole minutes only)

13 Colleen delivers junk mail and is paid \$32 to traverse a particular route and a further 10 cents per leaflet delivered.
  a What method of payment is Colleen being paid?
  b Write an equation for the total payment that she receives.
  c Sketch a graph of the relationship expressed in **b**.
  d What would Colleen's pay be if she delivered 1650 pamphlets?

14 A pay TV salesperson receives \$300 per week plus \$20 for every household that he signs up to have pay-TV connected. How much does he receive in a week where he signs up 33 households?

15 a Draw a graph of the relationship described in question **14**.
  b What is the point where the graph cuts the vertical axis?
  c How does this relate to the rate at which he is paid?

16 A person is running at 10 km/h. The speed at which she runs decreases by 1 km/h for every 30 minutes she has been running.
  a Draw a graph of the relationship between speed and the time that she has been running.
  b How long can she run before she will have to stop?

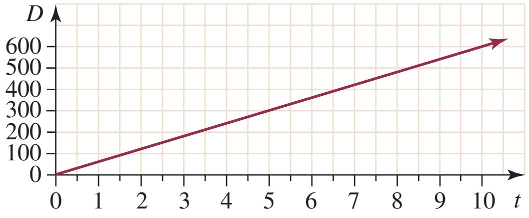
INVESTIGATE: Graph of height versus age

Not all graphs can be drawn as a straight line. Consider the case of height and age.

1. Find a person of each age from 1–20. Measure their height and plot their age and height as a pair of coordinates.
2. Draw a line of best fit for the points plotted.
3. The graph will flatten where people stop growing and so does not continue to rise indefinitely. Suggest a point at which this graph should stop.

7B Gradient and intercept

Consider the graph of a car that is travelling at 60 km/h. Earlier we drew the graph of this as a linear function.

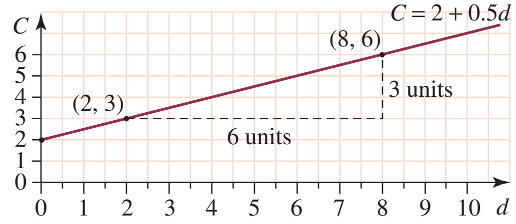


Two points on this graph are (1, 60) and (2, 120). From the graph we can see that for a one unit increase in the independent variable, there is a 60 unit increase in the dependent variable. For this function we can say that the gradient is 60.

The **gradient** (m) is the **rate of change** in the dependent variable for a one unit increase in the independent variable. A simple formula that can be used to calculate gradient is:

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Using this formula, the gradient can be calculated by measurement from a graph by choosing any two points on the graph.



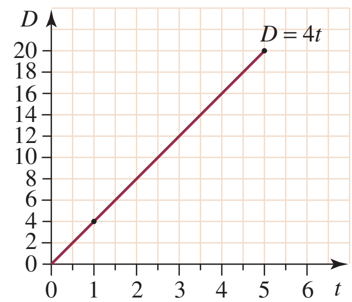
The graph at right shows the function $C = 2 + 0.5d$.

On the graph, the two points (2, 3) and (8, 6) are marked. Between these two points the vertical rise = 3 and the horizontal run = 6. Using the gradient formula:

$$\begin{aligned}\text{gradient} &= \tfrac{3}{6}\\ &= \tfrac{1}{2}\end{aligned}$$

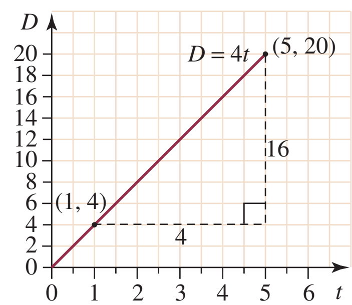
WORKED EXAMPLE 4

For the linear function drawn at right, calculate the gradient.



THINK | **WRITE**

1. Choose two points on the graph: (1, 4) and (5, 20) for example.
2. Measure the vertical rise and the horizontal run.



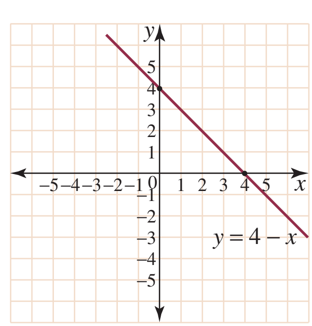
3. Write the gradient formula. $\text{gradient} = \dfrac{\text{vertical change in position}}{\text{horizontal change in position}}$
4. Substitute for the rise and the run. $= \frac{16}{4}$
5. Calculate the gradient. $= 4$

A function with a positive gradient is called an **increasing function**. That means that the value of the dependent variable increases as the value of the independent variable increases.

A **decreasing function** has a negative gradient. In such cases when calculating the gradient, we take the vertical rise to be negative. In a decreasing function, the value of the dependent variable decreases as the value of the independent variable increases.

WORKED EXAMPLE 5

For the function drawn at right calculate the gradient.



THINK

1 Choose two points on the graph. In this case we choose (0, 4) and (4, 0).

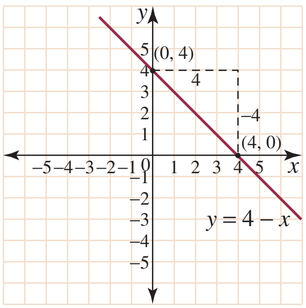
2 Measure the vertical rise and the horizontal run.

3 Write the gradient formula.

4 Substitute for the rise and the run.

5 Calculate the gradient.

WRITE



$$\text{gradient} = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

$$= \frac{^-4}{4}$$

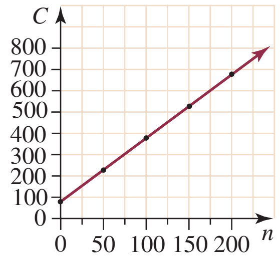
$$= {}^-1$$

The gradient of −1 in the example above means that for every one-unit increase in x, there is a one unit decrease in y.

In the example above, the graph cuts the y-axis at 4. Therefore, for this function the ***y*-intercept** is 4.

Consider worked example 3. Here the cost of hiring the entertainment group was given by the function $C = 80 + 3n$.

In this example, the intercept on the vertical axis is 80; that is, it costs $80 to hire the entertainment group without any children attending the concert. The $80 is a fixed cost.



WORKED EXAMPLE 6

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Worked example 6

The table below shows the cost of running an excursion for a given number of students.

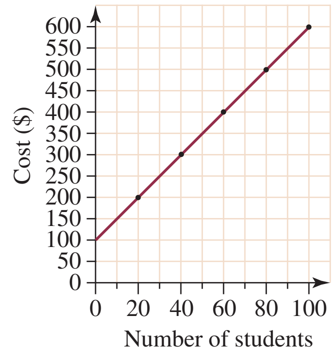
No. of students	20	40	60	80	100
Cost	$200	$300	$400	$500	$600

a **Draw a graph of the cost of this excursion.**
b **Calculate the gradient and explain its meaning in this context.**
c **Use your graph to find the intercept on the vertical axis and explain its meaning in this context.**

THINK | WRITE/DRAW

a 1 Draw a set of axes and plot the points given.

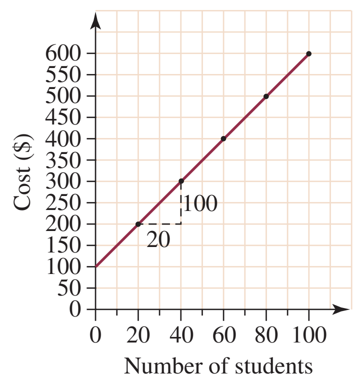
a



2 Join with a straight line.

b 1 Choose two points on the graph and measure the vertical change in position and horizontal change in position.

b



2 Calculate the gradient.

$$\text{gradient} = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$
$$= \frac{100}{20}$$
$$= 5$$

3 The gradient is the increased cost of the excursion per student.

A gradient of 5 means that the cost of the excursion increases by $5 for every student who attends.

c 1 Find the point where the graph cuts the vertical axis.

c Intercept = 100

2 The intercept is the fixed cost of running an excursion without considering the number of students.

The excursion has a fixed cost of $100, meaning it would cost $100 even if no children attended.

REMEMBER

1. The gradient is the increase in the dependent variable for every one unit increase in the independent variable.
2. The gradient is denoted m, and is found using the formula:

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

3. A positive gradient occurs when the value of the dependent variable increases as the value of the independent variable increases.
4. A negative gradient occurs when the value of the dependent variable decreases as the independent variable increases.
5. The intercept on the vertical axis gives us the value of the dependent variable when the independent variable is equal to zero.

EXERCISE
7B Gradient and intercept

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Gradient of a straight line

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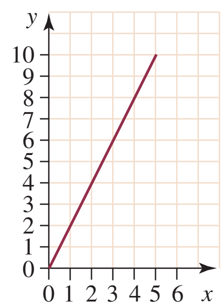
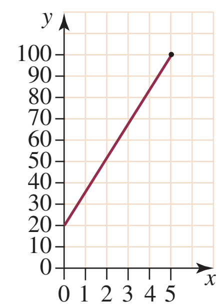
Gradient

1 WE4, 5 For the functions below, find the gradient.

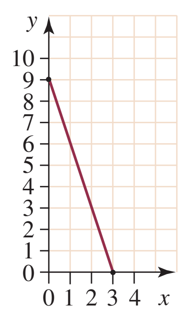
a



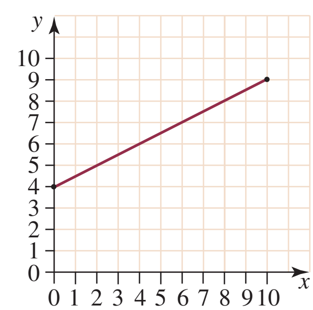
b



c



d



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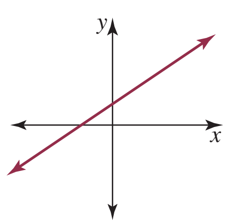
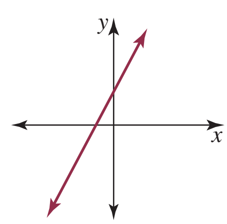
Graph paper

2 MC Which of the functions below has a negative gradient?

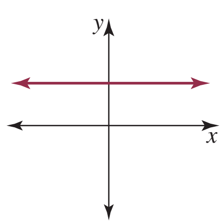
A



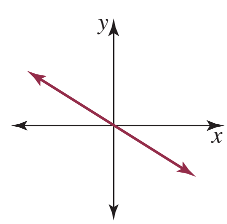
B



C



D



3 WE6 The table below shows the payment made to a person on a newspaper delivery round.

Deliveries	200	400	600	800	1000
Payment	120	180	240	300	360

a Draw the graph of the function.
b Find the gradient of the function.
c Find the intercept on the vertical axis.

4 The table below shows the profit or loss made by a cinema for showing a movie.

No. of people	Profit
20	–60
50	0
100	100
150	200
200	300

a Draw the graph of the function.
b Find the gradient of the function. Explain the meaning of the gradient in this context.
c Find the intercept on the vertical axis. Explain its meaning in this context.

5 A function is given by the rule $y = 5x - 4$.
a Copy and complete the table below.

x	0	1	2	3
y				

b Draw the graph of this function.
c Find the gradient and intercept of this function.

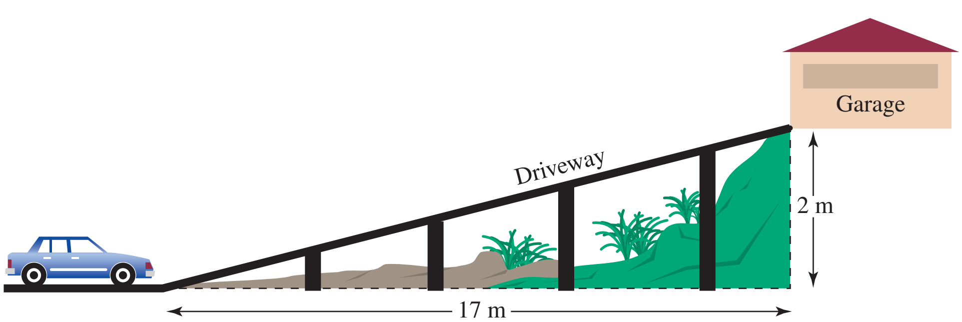
Further development

6 a Sketch two different graphs that have the same gradient.
b What can be concluded about lines that have the same gradient?

7 a Sketch a graph that has a gradient of zero.
b What can be said about lines with a zero gradient?

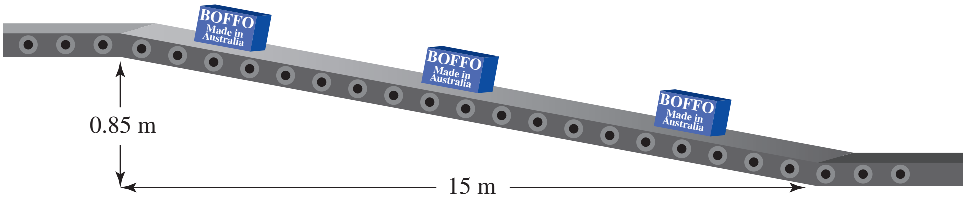
8 a Sketch a vertical line.
b Why is it not possible to find the gradient of a vertical line?

9 Bughar plans the construction of a proposed driveway on a plan which is below.



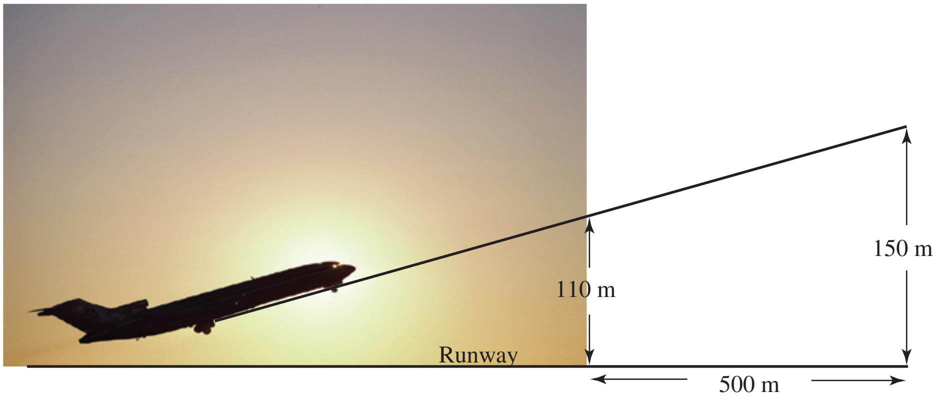
What is the gradient of the proposed driveway?

10 An assembly line is pictured below.



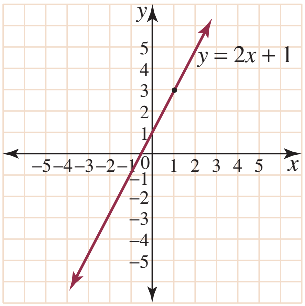
What is the gradient of the sloping section? Give your answer as a simplified fraction.

11 A passenger jet takes off on the following path.



What is the gradient of the planes ascent?

7C Drawing graphs using gradient and intercept



Most linear functions are represented on a number plane. Consider the graph of $y = 2x + 1$ drawn at right.

This function has a gradient of 2. The intercept on the vertical axis (called the *y*-intercept) is 1. Comparing the gradient and *y*-intercept with the function, we can see that the number with *x* (called the coefficient of *x*) is 2 (the gradient) and we then add 1 (*y*-intercept) to complete the function.

Any linear function can be written in the form $y = mx + b$ where m = gradient and b = *y*-intercept.

WORKED EXAMPLE 7

Find the gradient and *y*-intercept of:

a $y = 3x - 4$ **b** $y = 7 - 2x$.

THINK	WRITE
a 1 The gradient is the coefficient of *x* (3).	**a** gradient = 3
2 The *y*-intercept is the constant term (–4).	*y*-intercept = –4
b 1 The gradient is the coefficient of *x* (–2).	**b** gradient = –2
2 The *y*-intercept is the constant term (7).	*y*-intercept = 7

We can use the gradient and y-intercept to draw the graph of a function in the form $y = mx + b$. By plotting the y-intercept we are able to use the gradient to plot other points. For example, a gradient of 2 means a rise of 2 units for a 1 unit increase in x. Therefore, from the y-intercept we count a rise of 2 units and a run of 1 unit to plot the next point. It is a useful check to repeat this process from the next point plotted. The points plotted can then be joined by a straight line that is the graph of the function.

WORKED EXAMPLE 8

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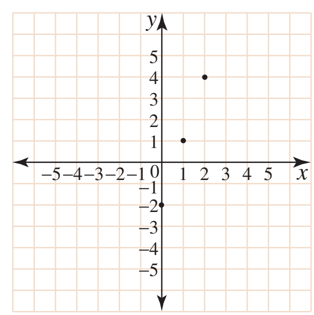
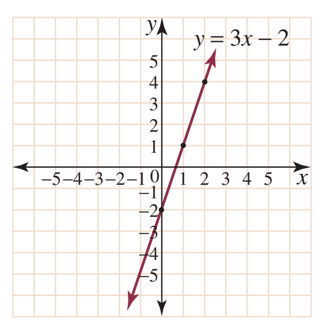
Tutorial
int-2399
Worked example 8

Draw the graph of $y = 3x - 2$.

THINK	WRITE
1 Find the gradient (3).	gradient = 3
2 Find the y-intercept (–2).	y-intercept = –2
3 Mark the y-intercept on the axis.	
4 Count a rise of 3 and a run of 1 to mark the point (1, 1).	
5 From (1, 1) count a rise of 3 and a run of 1 to mark the point (2, 4).	
6 Join these points with a straight line.	

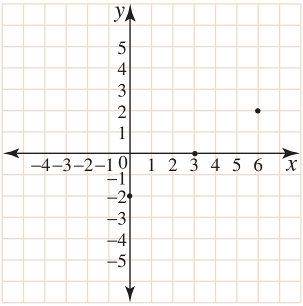
If the gradient is a fraction, the numerator indicates the vertical change in position and the denominator the horizontal change in position. The method of drawing the graph then remains unchanged.

WORKED EXAMPLE 9

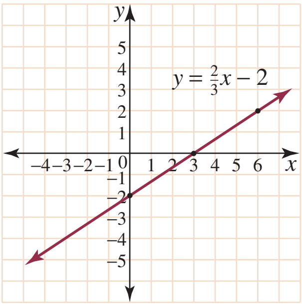
Sketch the graph of $y = \frac{2}{3}x - 2$.

THINK	WRITE
1 Find the gradient $(\frac{2}{3})$.	gradient = $\frac{2}{3}$
2 Find the y-intercept (–2).	y-intercept = –2

3. Mark the y-intercept on the axis.
4. Count a rise of 2 and a run of 3 to mark the point (3, 0).
5. From (3, 0) count a rise of 2 and a run of 3 to mark the point (6, 2).



6. Join these points with a straight line.



When sketching functions with a negative gradient we need to remember to treat the rise as negative; that is, the function decreases.

WORKED EXAMPLE 10

Sketch the function $y = 3 - 2x$.

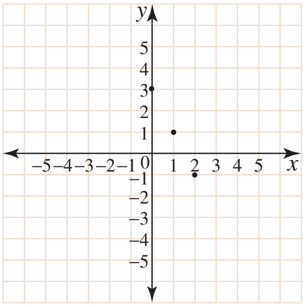
THINK

1. Find the gradient (–2).
2. Find the y-intercept (3).
3. Mark the y-intercept on the axis.
4. Count a rise of –2 and a run of 1 to mark the point (1, 1).
5. From (1, 1) count a rise of –2 and a run of 1 to mark the point (2, –1).

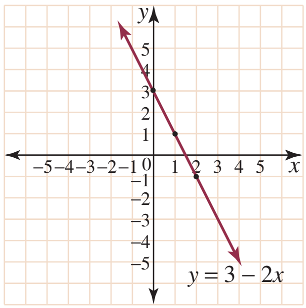
WRITE

gradient = –2

y-intercept = 3



6 Join these points with a straight line.



REMEMBER

1. A function is written in the form:

$$y = mx + b$$

where m equals the gradient and b equals the y-intercept.

2. A function can be graphed when in this form by plotting the y-intercept, then using the gradient to plot two other points, which can then be joined with a straight line.

EXERCISE 7C Drawing graphs using gradient and intercept

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Digital doc
EXCEL Spreadsheet doc-1532
Equation of a straight line

1 WE7 For each of the functions below, state the gradient and the y-intercept.

a $y = 2x + 2$ **b** $y = 3x - 8$ **c** $y = 2 - 4x$

d $y = \frac{3}{4}x + 3$ **e** $y = \frac{x}{2} + 1$ **f** $y = 3 - \frac{3}{2}x$

2 WE8 Sketch the function $y = 2x - 3$.

3 Sketch the functions:

a $y = 2x + 1$ **b** $y = 3x - 6$ **c** $y = 5x$.

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Digital doc
EXCEL Spreadsheet doc-1533
Linear graphs

4 WE9 Sketch the function $y = \frac{1}{2}x + 2$.

5 Sketch the graph for each of the functions below.

a $y = \frac{3}{4}x - 1$ **b** $y = \frac{1}{3}x$ **c** $y = \frac{3}{2}x - 4$

6 WE10 Sketch the function $y = 4 - 3x$.

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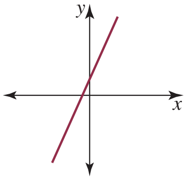
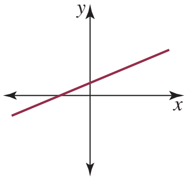
Digital doc
EXCEL Spreadsheet doc-1534
Graph paper

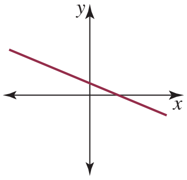
7 Sketch the graphs of:

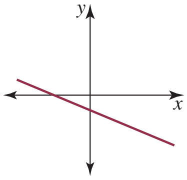
a $y = 6 - 3x$ **b** $y = -2x - 3$ **c** $y = -\frac{1}{2}x + 4$.

8 MC Which of the following could be the graph of $y = -\frac{1}{2}x + 1$?

A


B


C


D


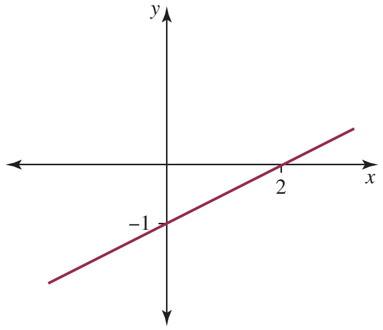
9 MC The equation of the graph drawn below could be:

A $y = 2x - 1$

B $y = 2x + 1$

C $y = \frac{1}{2}x - 1$

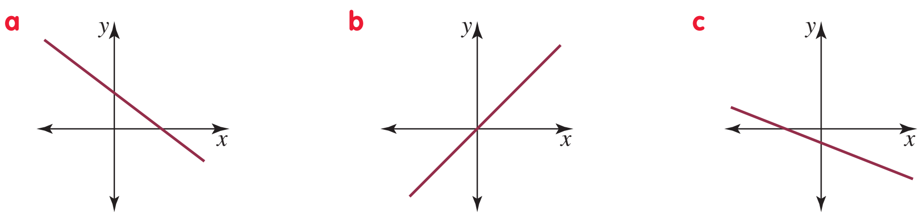
D $y = \frac{1}{2}x + 1$



10 Write and draw an example of a linear function with:

a a positive gradient
b a negative gradient
c a positive y-intercept
d a y-intercept of 0
e a negative gradient and negative y-intercept
f a gradient of 0
g a positive gradient and negative y-intercept.

11 Write an equation that *could* fit the following sketches.



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WorkSHEET 7.1
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Further development

12 a How is a graph with a negative gradient recognised?
b How do you know that $y = 5 - 2x$ has a negative gradient?

13 a How is a graph with a positive vertical intercept recognised?
b Does the graph of $y = 5 - 2x$ have a positive or negative vertical intercept?

14 By making y the subject of the formula $3x - y = 10$ find:
a the gradient
b the vertical intercept.

15 What would be the equation of a line that has:
(write your answer without the use of fractions)
a gradient = 3 and y-intercept = 4
b gradient = –1 and y-intercept = –4
c gradient = $-\frac{1}{2}$ and y-intercept = 1.

16 Determine the geometrical similarity between $y = 2x - 1$ and $2x - y - 5 = 0$.

17 a Find the gradient of $y = 2 - \frac{1}{2}x$ and $2x - y - 5 = 0$
b What is the geometrical significance of these two lines?

7D Graphing variations

A variation occurs when one quantity is **proportional to** another. Consider the following variation problem.

The number of cars produced on an assembly line varies directly with the number of workers employed on the line. Twenty workers can produce 30 cars per week.

From this information, we can determine that the number of cars produced each week will be 1.5 times the number of workers employed on the assembly line. Using this, we can draw the table below.

No. of workers (N)	No. of cars produced (C)
10	15
20	30
30	45
40	60
50	75

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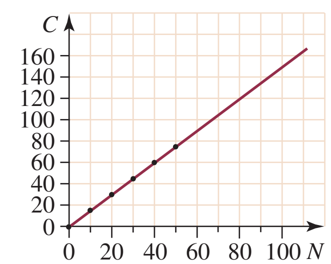
Interactivity
int-2399
Slope and equation of a line

These figures are plotted on the axes at right.

In any example where one quantity varies directly with another, the graph that is drawn will be a linear function through the origin (0, 0). To draw the function, we need to know only one other point on the graph. This is known as a **direct linear variation**.



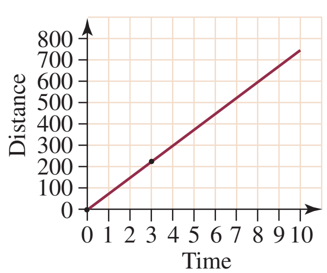
WORKED EXAMPLE 11

The distance travelled by a car is directly proportional to the speed at which it is travelling. If the car travels 225 km in 3 hours, draw a graph of distance travelled against time.

THINK

1. Draw a set of axes showing time on the horizontal axis and distance on the vertical axis.
2. Plot the points (0, 0) and (3, 225).
3. Join them with a straight line.

WRITE



If we examine the gradient of a variation function, we see that the gradient is equal to the *constant of variation*. For example in worked example 11, the gradient is 75. This is the speed at which the car is travelling.

Any variation can be graphed using the form $y = ax$ where a, the gradient, is also the constant of variation.

WORKED EXAMPLE 12

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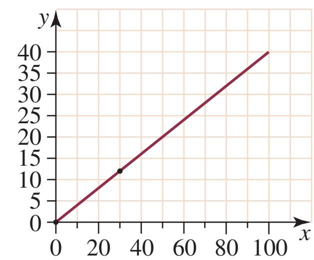
Tutorial
int-2317
Worked example 12

It is known that y is directly proportional to x. When $x = 30$, $y = 12$.
a Draw the graph of y against x.
b What is the gradient of the graph?
c Write an equation linking y and x.

THINK	WRITE
a Draw a straight line graph through (0, 0) and (30, 12).	**a** 
b 1 $\text{Gradient} = \dfrac{\text{vertical change in position}}{\text{horizontal change in position}}$	**b** $\text{Gradient} = \frac{12}{30}$
2 Simplify.	$= 0.4$
c The equation is in the form $y = ax$, where a is the gradient.	**c** $y = 0.4x$

REMEMBER

1. When two quantities vary directly with each other, the variation can be graphed as a linear function.
2. To graph the function we need to know only one point on the graph, together with (0, 0). A straight line is then drawn through these two points.
3. The gradient of the function is the constant of variation. Hence, the variation is graphed using a linear function that can be written in the form $y = ax$.

EXERCISE 7D Graphing variations

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Digital doc
SkillSHEET 7.3
doc-1535
Substitution

eBook*plus*

Digital doc
SkillSHEET 7.4
doc-1536
Graphing linear equations

1 WE11 The distance travelled by a car varies directly with the time that the car has been travelling. If the car travels 400 km in 5 hours, draw the graph of distance against time.

2 A team of 6 people can unload 9 containers from a wharf per day.
- **a** Draw a graph showing the number of containers, n, that can be unloaded by a team of people, p.
- **b** What is the gradient of the graph drawn?
- **c** Write an equation linking n and p.

3 WE12 It is known that y varies directly with x. When $x = 5$, $y = 40$.
- **a** Draw the graph of y against x.
- **b** What is the gradient of the graph?
- **c** Write an equation linking y and x.

4 The distance, D, travelled by a car in a certain period of time will be directly proportional to the car's speed, s. A car moving at 40 km/h travels 120 km.
- **a** Draw the graph of D against s.
- **b** Write an equation linking D and s.

5 The wage, W, earned by a worker is directly proportional to the hours, h, worked. A person who works 35 hours earns \$306.25. Draw the graph of W against h.

6 The quantity of petrol, l, used by a car varies directly with the distance, d, travelled by the car. A car that travels 100 km uses 12.5 L of fuel.

a Draw the graph of l against d.

b Use the graph to find the quantity of petrol needed to travel 240 km.

7 The height of a tree, h, is directly proportional to the girth, g. A tree with a girth of 2.5 m has a height of 14 m.

a Draw the graph of h against g.

b Use the graph to find the height of a tree with a girth of 3 m.

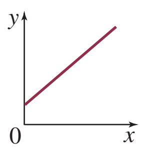
c Use the graph to find the girth of a tree that is 9 m tall, correct to 1 decimal place.

8 It is known that A\$100 will buy US\$67.50. Draw a conversion graph between Australian and US dollars.

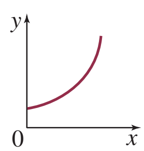
Further development

9 For each of the following, state if the graph could be a direct variation. For those which are not give a reason.

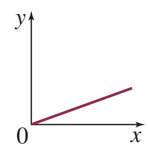
a



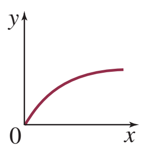
b



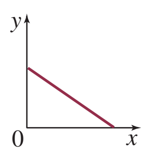
c



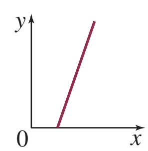
d



e



f



10 The directions on a bottle of kitchen mould remover recommend that you dilute half a cup of the concentrate in 5 litres of warm water.

a Complete the following table.

Volume of water (L)	1	2	3	4	5	10	15	20	30
Amount of cleaner (cups)									

b When graphed what will be the gradient of the variation?

c How does this relate to the mixture of concentrate and water?

11 The perimeter of a certain shape is directly proportional to the side length. When $P = 15$, $s = 3$.

a Find the perimeter of the shape if the side length is 6.2 cm.

b Find the length of the side when the perimeter is 67 cm.

c Name the shape.

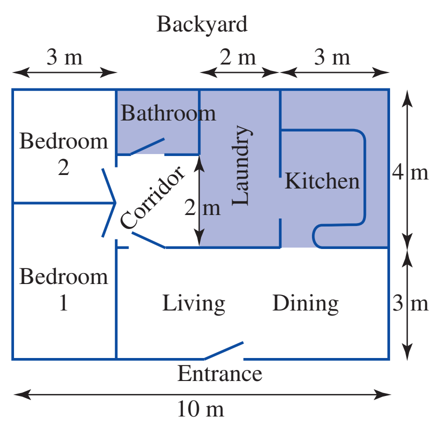
(*Hint:* You may or may not need to draw the graph to answer this question.)

12 Mika is going to polish all the floors in her unit, except for the kitchen, laundry and bathroom, where she has tiles. The plan of Mika's unit is shown below.

a Find the area that is to be polished.

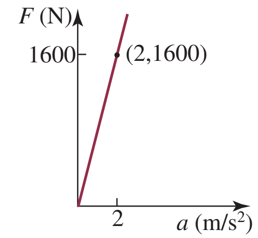
b A particular type of varnish is sold in 3 L cans. If one can covers 17.25 m^2 of flooring, how many cans of varnish will Mika need to purchase in order to do the floors twice?

c How much varnish will be wasted?



13 The graph below shows the relationship between acceleration (a) of a certain body and force (F) acting on that body.

a Find the gradient of the graph.
b Write the equation of the relationship.
c Find the force necessary to produce an acceleration of 4 m/s^2
d Find the acceleration required to produce a force of 1000 N.



14 If $(2, a)$ and $(a, 8)$ are points on a direct variation find:

a the value of a
b the constant of variation (the gradient of the graph).

INVESTIGATE: Currency conversions

Find out the current rate of conversion for each of the following foreign currencies and draw a linear function that will convert between Australian dollars and each currency.

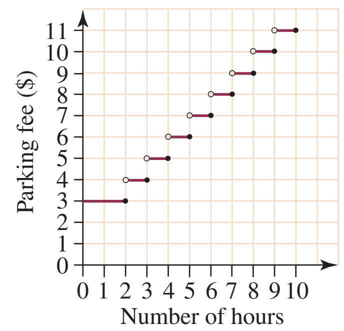
1 US dollars
2 Euro
3 Pound Sterling
4 Japanese yen
5 New Zealand dollars

7E Step and piecewise functions

A **step function** is a linear function for which the rule changes as the value of the independent variable changes.

Consider the case of a parking lot. The charge to park is $3 for the first 2 hours and $1 per hour after that.

The graph for the parking charges is shown at right. The graph is called a step graph because it looks like a staircase.



WORKED EXAMPLE 13

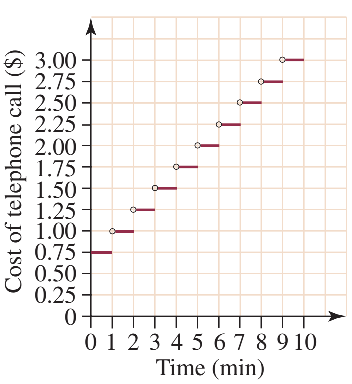
A telephone call is charged at 75c for the first minute and 25c per minute after that. Draw a graph of the cost of the telephone call.

THINK

1 Draw axes with time on the horizontal axis and cost on the vertical axis.

2 Draw a step function at 75c with increases of 25c every minute.

DRAW



A **piecewise linear function** is similar to a step graph. A piecewise function consists of more than one piece. In such examples we draw each linear function separately for the section of the graph to which it applies.

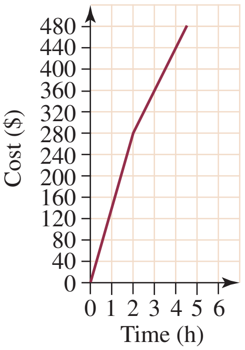
WORKED EXAMPLE 14

A catering company charges $140 per hour for the first 2 hours and $80 per hour thereafter. Show this as a piecewise linear graph.

THINK

1. Draw axes with time on the horizontal axis and cost on the vertical axis.
2. For the first 2 hours draw the graph at $140 per hour.
3. Three hours will cost $360, 4 hours $440. Plot these points and join them with a straight line.

DRAW



REMEMBER

1. A step function shows the increase in a quantity in steps.
2. A piecewise function follows different rules for different values of the independent variable.

EXERCISE 7E Step and piecewise functions

1 WE13 The cost of a bus fare is $1.20 for one section and 40c per section thereafter. Show this in a step graph.

2 The cost of posting a parcel is shown in the table below.

Mass	500 g or less	500 g to 2 kg	Over 2 kg
Cost	$2.50	$3.75	$5.50

Draw this information in a step graph.

3 A mobile telephone plan has a base charge of $25 per month, which includes 10 free calls. Every call thereafter costs $1.50. Show this information in a step graph.

4 WE14 A cyclist rides at an average 9 km/h uphill for 2 hours and then at 15 km/h for the next 3 hours. Draw a graph of distance travelled against time.

5 The cost of having business cards printed is $100 plus 50c each for the first 100, then 20c each thereafter. Draw a graph showing the cost of having these business cards printed.

6 Chandra earns $12 per hour for the first 35 hours worked each week. Any overtime is paid at time-and-a-half. Draw a piecewise graph that will show Chandra's pay.

7 The PAYG tax rates in Australia from several years ago are shown below.

Income	Tax payable
\$1 to \$6000	\$0
\$6001 to \$25 000	15% of each \$1 over \$6000
\$25 001 to \$75 000	\$2850 plus 30% of each \$1 over \$25 000
\$75 001 to \$150 000	\$17 850 plus 40% of each \$1 over \$75 000
In excess of \$150 000	\$47 850 plus 45% of each \$1 over \$150 000

Draw a piecewise function showing the amount of tax payable on income.

Further development

8 A computer repair company charges the following rates.
\$45 call out fee plus
\$35 per half hour or part thereof

a Calculate the charge for the following service calls.

i 20 min **ii** 45 min **iii** 80 min **iv** 90 min

b Draw a graph of charges (C) against the time (t) of the service call.

9 A mobile phone company charges users a rate of 20 cents for each completed 30 seconds of the call. This means that calls of less than 30 seconds are free. Draw the graph of cost versus call length for calls of up to 150 seconds

10 A real estate agent is paid a commission of 1.5% on the first \$400 000 and 1% on the remainder.

a Find the commission payable on a sale of

i \$300 000 **ii** \$500 000.

b Draw a graph of commission against sales.

c Explain the difference in gradients between the two sections.

11 The table below shows the rate at which electricity is charged per kWh (kilowatt hour).

Draw the graph of the rate at which electricity is charged.

Power	Cents per kWh
First 400 kWh	40
Next 1000 kWh	30
Balance	20

12 A hotel charges the following rates for a one week holiday package:

1 person \$1800
2 people \$3300
each extra person \$1200.

a Explain why the number of people on the holiday is a discrete variable.

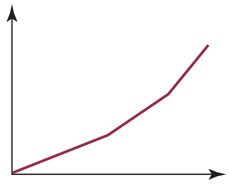
b Find the cost for:

i 1 person **ii** 2 people **iii** 3 people **iv** 5 people **v** 10 people.

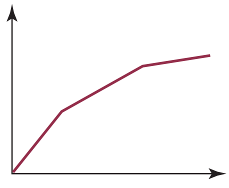
c As the data is discrete the cost is graphed using only a dot on each point. Graph the cost of the holiday for up to 10 people.

13 Identify a situation that could be represented by each of the following graphs.

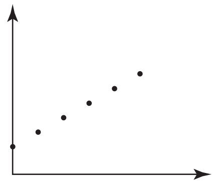
a



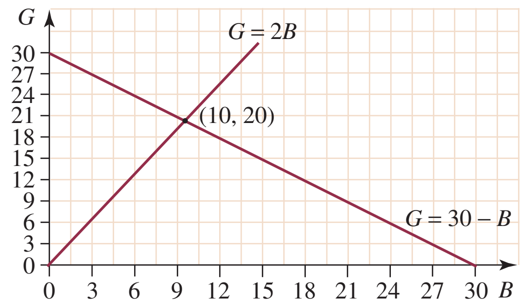
b



c



7F Simultaneous equations



Consider the problem below.

A class has 30 students. There are twice as many girls as boys. How many boys and girls are in this class?

We solve this problem by modelling two linear relationships.

We can say that $G = 30 - B$ and $G = 2B$, where G represents the number of girls and B represents the number of boys.

The solution to the problem will be the point of intersection of these linear relationships.

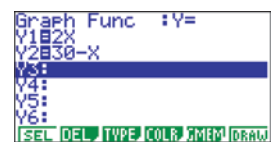
The point of intersection on these lines is (10, 20). Therefore the solution to this problem is 10 boys and 20 girls.

Graphics Calculator tip! Finding the point of intersection

Once you have graphed two functions, the point of intersection can be found using your graphics calculator. Consider the functions drawn previously.

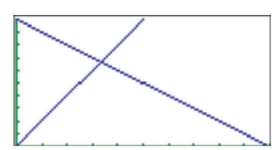
1. From the **MENU** select **GRAPH**.

2. Delete any existing functions and enter the functions **Y1 = 2X** and **Y2 = 30 – X**.

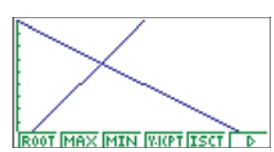


3. Press SHIFT F3 for **V-Window**. Enter the settings shown at right, which match the axes drawn above.

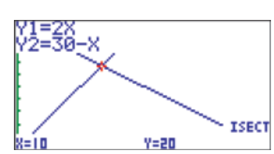
4. Press EXE to return to the previous screen, and then press F6 to draw the graphs.



5. To find the intersection press SHIFT F5 for **G-Solv** (a graph-solving function).



6. Press F5 for **ISCT** to find the intersection of the two graphs. This may take a moment for the calculator to find.



WORKED EXAMPLE 15

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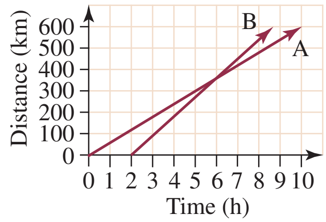
Tutorial
int-2318
Worked example 15

Car A is travelling at a constant speed of 60 km/h. Car B leaves 2 hours later and travels at a constant speed of 90 km/h. This is represented by the linear model below.

How far from the starting point does car B overtake car A?



THINK	WRITE
1 Look for the point of intersection of the two graphs.	Point of intersection (6, 360).
2 Read the distance of this point on the y-axis.	Car B overtakes car A 360 km from the starting point.

REMEMBER

The point of intersection of two linear models will give the point where both conditions hold true.

EXERCISE 7F Simultaneous equations

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Digital doc
SkillSHEET 7.5
doc-1538
Solving linear equations

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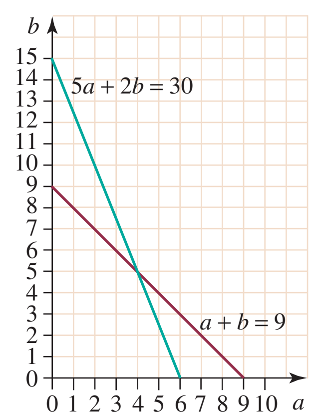
Digital doc
EXCEL Spreadsheet
doc-1539
Simultaneous equations

1 WE15 At the grocery store, apples cost \$5 per kg and bananas cost \$2 per kg. Rhonda spends \$30 on 9 kg of fruit. This can be represented by the linear functions at right, where a represents the number of apples and b represents the number of bananas.

Use the graph to find the mass of apples and bananas that Rhonda bought.



2 a Sketch the graphs of $y = 2x + 1$ and $y = 7 - x$.
b Write down the point of intersection of the two graphs.

3 A rectangle has a length of x and a width of y.
 a The perimeter of the rectangle is 22 cm. This can be represented by the linear function $2x + 2y = 22$. Graph this function.
 b The length of the rectangle is 5 cm longer than the width. This can be represented by the linear function $y = x - 5$. On the same set of axes graph this function.
 c Use the intersection of the two graphs to determine the length and width of the rectangle.

4 a Steve earns twice as much money each week as Theo. This can be represented by the linear function $s = 2t$, where s represents the amount of money Steve earns and t represents the amount of money Theo earns. Draw the graph of this function.
 b The total of Theo and Steve's wages is \$750. This can be represented by the linear function $t + s = 750$. Draw this function on the same set of axes.
 c Use the intersection of these graphs to find Theo's wage and Steve's wage.

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Digital doc
WorkSHEET 7.2
doc-1540

5 The sum of Tanya's English and Maths marks is 135.
 a Write a linear function that represents this information and sketch the function.
 b Tanya's English mark was 21 marks higher than her Maths mark. Write a linear function to represent this piece of information and draw the graph on the same set of axes.
 c Use the intersection of your two graphs to find Tanya's mark in both English and Maths.

Further development

6 A computer firm, SuperComputers Inc., offers a back-up plan covering the ongoing service and troubleshooting of its systems after sale. The cost of signing up for the service plan is \$215, and there is an hourly rate of \$65 for the serviceperson's time. Purchasers not signing up for the plan are charged a flat rate of \$150 per hour for service. Would it be advisable to sign up for the service plan if you expected to need 3 hours of service assistance during the life of a computer purchased from SuperComputers Inc?

7 A telephone company, Opus, offers calls to Biddelonia for a connection fee of \$14, and \$1 per minute thereafter. Its rival, Belecom, offers calls for \$2 per minute (no connection fee) to the same country.
 a Compare the cost of a 10 minute call to Biddelonia using each company.
 b At what point would it be cheaper to use Opus?

8 It costs you \$6 to get into a taxi (the 'flagfall'), and \$1.50 per kilometre if you use 'PinkCabs', while NoTop taxis charge \$8 flagfall, and \$1.20 per kilometre.
 a How much would it cost with each company to travel 15 km in one of its cabs?
 b When would it cost the same to use both companies?

9 Medirank, a health insurance company, charges \$860 per year (for a single person), and requires customers to pay the first \$100 of any hospital visit. HAB, on the other hand, charges an annual fee of \$560 and requires its members to pay the first \$150 of any hospital visit. Determine the number of hospital visits in a year for which the cost of health services is the same for either company.

10 Nifty is a car hire firm that charges insurance of \$135, and \$50 per day car hire. A competitor, Savus, simply charges \$65 per day and offers 'free' insurance. You are planning a holiday and would prefer to use Savus. Under what conditions (days hired) could you justify this choice?

SUMMARY

Graphing linear functions

- A function is a rule for calculation that involves an independent variable and a dependent variable.
- If the function is a straight line when graphed, then the function is called a linear function.
- The independent variable is shown on the horizontal axis and the dependent variable on the vertical axis.
- The function can be graphed by drawing a table of values and then plotting the points generated, joining them with a straight line.

Gradient and intercept

- The gradient is the increase in the dependent variable per one unit increase in the independent variable.
- The gradient (m) can be found using the formula

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}.$$

- If the function is decreasing, then the gradient will be negative.
- The intercept on the vertical axis gives the value of the dependent variable when the independent variable is equal to zero.
- A function is written in the form:

$$y = mx + b$$

  where m equals the gradient and b equals the y-intercept.
- The gradient and the y-intercept can be used to help draw the graph of a function.

Graphing variations

- When two quantities vary directly with one another, the variation can be graphed as a linear function.
- The variation will be in the form $y = ax$.
- The graph is drawn from the point (0, 0) to one other point that is given.
- The gradient of the linear function will equal the constant of variation.

Step and piecewise functions

- A step function occurs when the value of the dependent variable increases in steps rather than continuously.
- A piecewise function occurs when the function has different rules applying for different values of the independent variable.

Simultaneous equations

- The point of intersection of two linear functions gives the point where both functions hold true simultaneously.
- This is known as solving simultaneous equations.

CHAPTER REVIEW

MULTIPLE CHOICE

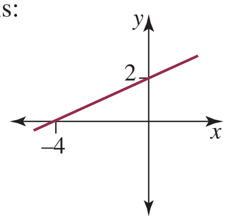
1 MC Look at the linear function drawn below. The gradient of this function is:

A -2

B $-\frac{1}{2}$

C $\frac{1}{2}$

D 2



2 MC The function $y = 6 - x$ has a gradient of:

A -6

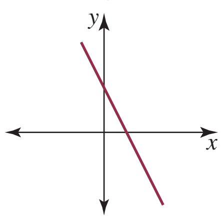
B -1

C 1

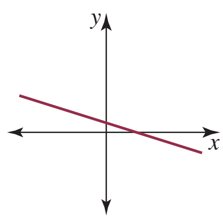
D 6

3 MC For which of the functions drawn below is the gradient the greatest?

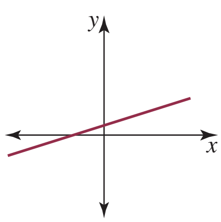
A



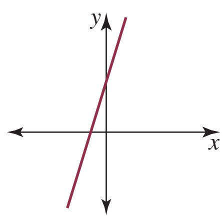
B



C



D



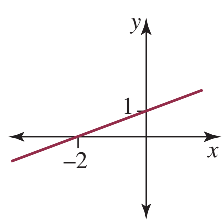
4 MC The linear function drawn below is the graph of:

A $y = \frac{1}{2}x - 1$

B $y = \frac{1}{2}x + 1$

C $y = 1 - \frac{1}{2}x$

D $y = \frac{1}{2}x - 2$



SHORT ANSWER

1 The table below shows the labour charge for working on a motor vehicle.

Hours (h)	1	2	3	4	5
Cost (C)	55	80	105	130	155

a Draw the graph of this function.

b Use the graph to find the labour charge for 8 hours work.

2 The conversion rate between Australian dollars, A, and Euro, E, can be defined by the rule $E = 0.7A$. Draw the linear function that will convert between the two currencies.

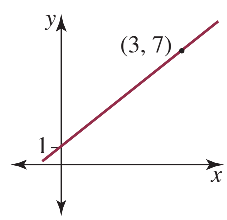
3 The cost, C, of having a parcel delivered by courier is given by the linear function $C = 20 + 3k$, where k is the number of kilometres the parcel must be delivered.

a Draw a graph of this function.

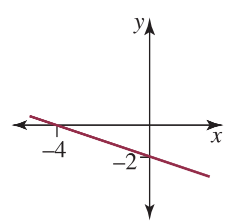
b Use the graph to determine the cost of having the parcel delivered a distance of 12 km.

4 For the functions below, find the gradient and vertical intercept.

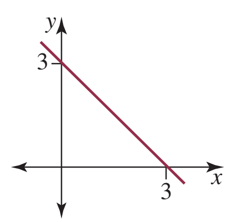
a



b



c



5 The table below shows the profit or loss that would be made from a cake stall given the number of cakes sold.

Number	20	40	60	80	100
Profit	–30	20	70	120	170

a Draw the graph of this function.

b State the gradient of this function. State the meaning of the gradient in this context.

c State the vertical intercept for this function. State the meaning of the vertical intercept in this context.

6 For each of the linear functions below, state the gradient and the y-intercept.
 a $y = 3x - 2$
 b $y = \frac{3}{4}x + 7$
 c $y = 5 - x$

7 Sketch each of the functions shown below.
 a $y = 2x - 1$
 b $y = 6 - 3x$
 c $y = \frac{1}{2}x + 3$

8 The cost of a tank of petrol varies directly with the amount of petrol purchased. If 25 L of petrol costs \$21.25, graph the variation.

9 It is known that p varies directly with q. When $p = 5$, $q = 15$.
 a Draw the graph of q against p.
 b What is the gradient of the graph?
 c Write an equation linking p and q.

10 The table below shows the cost per minute of a long distance telephone call.

Distance of call	Cost per minute
Less than 150 km	30c
150 km–750 km	65c
Over 750 km	90c

Show this information in a step graph.

11 A tree has an initial height of 75 cm. The growth rate of the tree is approximately 75 cm per year for the first 4 years, and 40 cm per year thereafter. Show this by way of a piecewise graph.

12 A rectangle has a length of l and a width of w.
 a The length of the rectangle is 10 cm longer than the width. This can be represented by the linear function $w = l - 10$. Draw this function.
 b The perimeter of the rectangle is 40 cm. This can be represented by a linear function. On the same set of axes draw this function. (*Hint:* Use the perimeter formula.)
 c Use the point of intersection of your two functions to find the length and the width of the rectangle.

EXTENDED RESPONSE

1 The table below shows the values of x and y in a linear function.

x	0	1	2	3	4
y	–3	–1	1	3	5

 a Plot the points and draw the graph of the linear function.
 b What is the gradient of the function?
 c What is the y-intercept?
 d Write the equation of this function.
 e On the same axes, draw the graph of $y = 5 - 2x$.
 f Write the solution to the pair of simultaneous equations represented on your diagram.

2 It is known that a quantity, m, varies directly with n. When $m = 2.5$, $n = 1.5$.
 a Draw a graph of n against m.
 b What is the gradient of the function?
 c What is the equation of this linear function?

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Test Yourself
doc-1541
Chapter 7

eBookplus ACTIVITIES

Are you ready?

Digital docs *(page 202)*
- SkillSHEET 7.1 (doc-1523): Recognising linear relationships
- SkillSHEET 7.2 (doc-1529): Gradient of a straight line
- SkillSHEET 7.3 (doc-1535): Substitution
- SkillSHEET 7.4 (doc-1536): Graphing linear equations
- SkillSHEET 7.5 (doc-1538): Solving linear equations

7A Graphing linear functions

Tutorial
- WE3 int-2315: Learn how to draw the graph of a linear relationship. *(page 205)*

Digital docs
- SkillSHEET 7.1 (doc-1523): Recognising linear relationships *(page 205)*
- Spreadsheet (doc-1531): Graph paper *(page 206)*
- GCprogram — Casio (doc-1525): Linear *(page 206)*
- GCprogram — TI (doc-1526): Linear *(page 206)*
- GCprogram — Casio (doc-1527): Myrule *(page 206)*
- GCprogram — TI (doc-1528): Myrule *(page 206)*

7B Gradient and intercept

Tutorial
- WE6 int-2316: Learn how to draw a linear graph from a table of values. *(page 209)*

Digital docs
- SkillSHEET 7.2 (doc-1529): Gradient of a straight line *(page 211)*
- Spreadsheet (doc-1530): Gradient *(page 211)*
- Spreadsheet (doc-1531): Graph paper *(page 211)*

7C Drawing graphs using gradient and intercept

Digital docs
- Spreadsheet (doc-1532): Equation of a straight line *(page 216)*
- Spreadsheet (doc-1533): Linear graphs *(page 216)*
- Spreadsheet (doc-1531): Graph paper *(page 216)*
- WorkSHEET 7.1 (doc-1523): Draw graphs of linear functions. *(page 217)*

7D Graphing variations

Interactivity
- int-2399: Slope and equation of a line. *(page 218)*

Tutorial
- WE12 int-2317: Learn to recognise and work with direct variation. *(page 219)*

Digital docs
- SkillSHEET 7.3 (doc-1535): Substitution *(page 219)*
- SkillSHEET 7.4 (doc-1536): Graphing linear equations *(page 219)*

7F Simultaneous equations

Tutorial
- WE15 int-2318: Learn how to determine apply simultaneous equations in everyday situations. *(page 225)*

Digital docs
- SkillSHEET 7.5 (doc-1538): Solving linear equations *(page 225)*
- Spreadsheet (doc-1539): Simultaneous equations *(page 225)*
- WorkSHEET 7.2 (doc-1540): Solve problem involving simultaneous equations. *(page 226)*

Chapter review

- Test yourself Chapter 7 (doc-1541): Take the end-of-chapter test to test your progress. *(page 229)*

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